

Multi-norms and multi-Banach algebras

There is a huge literature on operators on Banach spaces and on Banach algebras. (For background on Banach algebras, see my book ‘Banach algebras and automatic continuity’, published by Oxford University Press in 2000.) There is also a growing corpus on operator spaces.

We wish to introduce a new concept, related to, but somewhat different from, that of operator spaces.

Let $(E, \|\cdot\|)$ be a Banach space. A *multi-norm* is a sequence $(\|\cdot\|_n)$, where $\|\cdot\|_n$ is a norm on the space E^n for each $n \in \mathbb{N}$, $\|\cdot\|_1 = \|\cdot\|$ on E , and certain axioms are satisfied.

In the first lecture, I shall define these multi-norms, and give a variety of examples. In particular, we shall obtain various examples on the spaces $L^p(\Omega)$.

In lecture 2, I shall explain a strong connection with the theory of absolutely summing operators on a Banach space; we have results on the geometry of Banach spaces. I shall also describe a natural multi-norm on a Banach lattice.

In lecture 3, I shall discuss the theory of ‘multi-bounded linear operators’. This generalizes the theory of regular bounded operators on a Banach lattice.

In lecture 4, I shall discuss what we should mean by the ‘dual multi-normed space’, and prove various results.

In lecture 5, I shall define a ‘multi-Banach algebra’, and give a variety of examples involving group algebras and algebras of operators. Our theory gives new results about the cohomological classification of modules over group algebras, with connections to the theory of amenable locally compact groups.

This is a new theory; there are quite a few results, but many rather obvious questions are still to be sorted out; it seems to be a valuable approach that leads to solutions of some problems in classical Banach algebra theory, and may be useful in attacks on other open problems.

Printed notes and exercises will be available.

H. G. Dales, Leeds, August 08.